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Kumar, R. (2013). Secure Implementation in Production Economies. *Mathematical Social Sciences*, 66(3), 372–378. <https://doi.org/10.1016/j.mathsocsci.2013.08.004>

Published in:
Mathematical Social Sciences

Document Version:
Peer reviewed version

Queen's University Belfast - Research Portal:
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Accepted Manuscript

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PII: S0165-4896(13)00075-9

DOI: <http://dx.doi.org/10.1016/j.mathsocsci.2013.08.004>

Reference: MATSOC 1697

To appear in: *Mathematical Social Sciences*

Received date: 8 October 2011

Revised date: 12 August 2013

Accepted date: 19 August 2013

Please cite this article as: Kumar, R., Secure implementation in production economies.
Mathematical Social Sciences (2013), <http://dx.doi.org/10.1016/j.mathsocsci.2013.08.004>

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Secure Implementation in Production Economies*

Rajnish Kumar[†]

August 11, 2013

Abstract

This paper shows that, in production economies, the generalized serial social choice functions defined by [Shenker [23]] are securely implementable (in the sense of [Saijo et al. [22]]) and that they include the well-known fixed path social choice functions.

Keywords: Secure implementation, double implementation, serial social choice function, fixed path methods

1 Introduction

Secure implementation [Saijo et al. [22]] requires double implementation in dominant strategy equilibrium and Nash equilibrium by the same mechanism. Experimental testing [Cason et al. [3]] has shown that this concept works better than the traditional implementation concepts like implementation in dominant strategy equilibria and implementation in Nash equilibria. Since secure implementation is stronger than both of the concepts of implementation mentioned above, negative (impossibility) results abound in various environments with richer domains. Various examples of such environments are provided in [Saijo et al. [22]]. More recent negative results include Bochet and Sakai [1], and Fujinaka and Wakayama [9].

We look for secure implementability in production economies with divisible goods. In contrast to the negative results in various environments, we find that a very broad generalization of "serial" social choice function (SCF) [Moulin and Shenker [19]] as defined in [Shenker [23]] is securely implementable. We call such functions generalized serial SCFs (GSS). We also find that under certain mild conditions the fixed path SCFs are special cases of GSS and thus they are also securely implementable.

The intuition behind the idea that the serial SCF (or more generally the GSS) possesses the nice incentive property of secure implementability – whereas, as we will discuss later, the SCF corresponding to other well-known cost sharing rules like the Aumann-Shapley rule (which is the proportional rule in homogeneous goods case) does not share this feature¹ – is the following. In the latter, by changing the report, an agent can affect the outcome for all agents simultaneously. In particular, that agent's report changes the outcomes of such agents whose reports in turn, can change his outcome. This severe nature of externality in such SCF violates the acyclicity condition necessary for the combination of non-bossiness and strategyproofness (see, Satterthwaite and Sonnenschein [25]) of the SCF which in turn is necessary for secure implementability (Proposition 2 in Saijo et al. [22]). Under the serial SCF, on the contrary, protecting lower demanders² from the demands of higher demanders makes the externality one-sided and thus not so severe. More precisely, a change in the report of low demanders changes the outcomes for all high demanders whereas a small change in the report of high demanders doesn't affect the outcome for lower demanders.

The rest of the paper is arranged as follows. In section 2 we introduce the precise notion of secure implementability and state one proposition that characterizes the securely implementable SCFs. In section 3 we define the serial cost sharing method and introduce some generalizations considered in the literature, with a special focus on the cost sharing methods whose strategic properties have been studied. In section 4 we define serial SCF and GSS, in section 5 we present two of our main results, and in section 6 we conclude the paper with a conjecture. The main proofs are gathered in Appendix A.

*I would like to thank my advisor Herve Moulin for his continuous guidance. I also thank Simon Grant, Dipjyoti Mazumdar, Manipushpak Mitra, Anirban Kar, Justin Leroux, and Gaurab Aryal for their critical comments. Detailed comments of two referees and the associate editor have been instrumental in shaping this version.

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¹The SCF corresponding to the Aumann-Shapley rule is not even strategyproof

²By low demander in homogeneous case we mean an agent who gets smaller share of the output and pays lower level of input as the final outcome of the SCF. In heterogeneous case, different generalizations of serial mechanism rank the agents in orders based on different criteria.

2 Secure implementability

We consider an arbitrary set of *alternatives* A and a finite set of *agents* $N = \{1, 2, \dots, n\}$, where $n \geq 2$. Typical agents are represented by letters i, j etc. The preference relation of agent i over the set A is represented by *utility function* u_i . The set of *admissible utility functions* for agent i is denoted by U_i . The cartesian product of U_1, U_2, \dots, U_n is represented by U , i.e., $U \equiv \prod_{i \in N} U_i$. A typical element of U is a *utility profile* $u = (u_1, \dots, u_n)$, which is an n -tuple of utility functions – one for each agent. A *social choice function* (SCF) $f : U \rightarrow A$ is a function that associates every $u \in U$ with a unique alternative $f(u)$ in A . A *mechanism* (or a game form) $g : S \rightarrow A$ is a function that assigns to every $s \in S$ a unique element of A , where $S \equiv \prod_{i \in N} S_i$ and S_i is the strategy space of agent i .

Definition 1 The mechanism g is called a *direct revelation mechanism* associated with the SCF f if $S_i = U_i$ for all $i \in N$ and $g(u) = f(u)$ for all $u \in U$.

Sometimes we may refer to a direct revelation mechanism as the SCF if no confusion arises. When the strategies of agents $j \neq i$ are fixed at $s_{-i} \equiv (s_1, s_2, \dots, s_{i-1}, s_{i+1}, \dots, s_n)$, agent i can induce certain outcomes by choosing strategies from the set S_i . The set of such outcomes denoted by $g(S_i, s_{-i})$ is called the attainable set or the *opportunity set* of agent i at s_{-i} . More formally, $g(S_i, s_{-i}) \equiv \{b \in A \mid \exists s_i \in S_i \text{ and } g(s_i, s_{-i}) = b\}$. The set of alternatives that agent i with utility u_i ranks weakly below the alternative $a \in A$ is called the weak lower contour set for agent i with utility u_i at a and is denoted by $L(a, u_i)$. More formally, $L(a, u_i) \equiv \{b \in A \mid u_i(a) \geq u_i(b)\}$. Given the mechanism $g : S \rightarrow A$, the strategy profile $s \in S$ is a *Nash equilibrium* (NE) of g at $u \in U$ if $\forall i \in N$, $g(S_i, s_{-i}) \subseteq L(g(s), u_i)$. Let's denote by $N^g(u)$ the set of Nash equilibria of g at u .

Definition 2 The mechanism g implements SCF f in Nash equilibria if for all $u \in U$, (i) $\exists s \in N^g(u)$ st. $g(s) = f(u)$ and (ii) $\forall s \in N^g(u)$, $g(s) = f(u)$.

The SCF f is *Nash implementable* if there exists a mechanism that implements f in Nash equilibria. Given the mechanism $g : S \rightarrow A$, the strategy profile $s \in S$ is a *dominant strategy equilibrium* of g at $u \in U$ if $\forall i \in N, \forall \tilde{s}_{-i} \in S_{-i}$, $g(S_i, \tilde{s}_{-i}) \subseteq L(g(s_i, \tilde{s}_{-i}), u_i)$. Let's denote by $DS^g(u)$ the set of dominant strategy equilibria of g at u .

Definition 3 The mechanism g implements f in dominant strategy equilibria if for all $u \in U$, (i) $\exists s \in DS^g(u)$ st. $g(s) = f(u)$ and (ii) $\forall s \in DS^g(u)$, $g(s) = f(u)$.

The SCF f is *dominant strategy implementable* if there exists a mechanism that implements f in dominant strategy equilibria. We now introduce formally the concept of secure implementation, which requires the existence of a mechanism that implements the SCF in Nash equilibria as well as in dominant strategy equilibria.

Definition 4 The mechanism g securely implements the SCF f if for all $u \in U$, (i) $\exists s \in DS^g(u)$ st. $g(s) = f(u)$ and (ii) $\forall s \in N^g(u)$, $g(s) = f(u)$.

The SCF f is *securely implementable* (SI) if there exists a mechanism that securely implements f . Strategyproofness is a requirement on an SCF that truth telling by the agents is a dominant strategy under the direct revelation mechanism. More formally, the SCF f satisfies *strategy proofness* (SP) if, $\forall u \in U, \forall i \in N, \forall \tilde{u}_i \in U_i, u_i(f(u)) \geq u_i(f(\tilde{u}_i, u_{-i}))$. Another technical property on the SCF, introduced in Saijo et. al. [22], which together with strategyproofness characterizes secure implementability, is called the *rectangularity property* and is defined as following. The SCF f satisfies the *rectangularity property* (RP) if for all $u, \tilde{u} \in U$, if $u_i(f(\tilde{u}_i, \tilde{u}_{-i})) = u_i(f(u_i, \tilde{u}_{-i}))$ for all $i \in N$, then $f(\tilde{u}) = f(u)$. The following characterization due to Saijo et al. [22] will be used in one of our main results.

Proposition 1 (Saijo et. al. [22]): An SCF f is securely implementable if and only if f satisfies the SP and the RP.

3 Serial cost sharing methods

Serial cost sharing method (Moulin and Shenker [19]) was first introduced for an environment where the goods demanded by the agents are homogeneous or, in other words, the agents demand various quantities of the same good. Since our purpose here is to extend this method to more general settings, we will define the problem in an environment where each agent $i \in N$ demands $q_i \in [0, q^{\max}] \subset R_+ \cup \{\infty\}$ quantity³ of a personalized⁴ good i . Thus q_i , the i th component of vector $q \in R_+^N$, can be thought of as the demand for good i as well as the demand of agent i . The cost of serving these demands is $C(q)$, which must

³ q^{\max} can be ∞ .

⁴ In some of the more general models, e.g., [11], [12], each agent may demand quantities of some or all of the goods.

be divided among the agents; the cost share of agent i is given by $x_i(q; C)$. The preference of agent i is defined over R^2 , which is continuous, increasing in q_i , decreasing in x_i and the upper contour set is convex⁵. Let a concave utility function $u_i(q_i, x_i)$ represent the preference of agent i . For the homogeneous goods case, a homogeneous cost function is defined as following (see, e.g., [24]). A cost function C is homogeneous if there exists $c_0 : R_+ \rightarrow R_+$ such that $C(q) = c_0(q_N)$ where, $q_N = \sum_{i \in N} q_i$. Here the serial cost sharing method is defined as follows. Consider, without loss of generality, $q_1 \leq q_2 \leq \dots \leq q_n$. Define, $q^i = (q_1, q_2, \dots, q_{i-1}, q_i, q_i, \dots, q_i)$ then ,

$$x_i(q; C) = \frac{C(q^i)}{n+1-i} - \sum_{k=1}^{i-1} \frac{C(q^k)}{(n+1-k)(n-k)} \quad (1)$$

This method works as follows. Agent 1, with the lowest demand q_1 pays $1/n$ th of the cost of nq_1 . Agent 2, with the second lowest demand pays agent 1's cost share, plus $1/(n-1)$ th of the incremental cost from nq_1 to $q_1 + (n-1)q_2$. Agent 3, with the next lowest demand pays agent 2's cost share, plus $1/(n-2)$ th of the incremental cost from $q_1 + (n-1)q_2$ to $q_1 + q_2 + (n-2)q_3$. And so on. This method is characterized by "anonymity" and "invariance of the cost share of low demanders by a change in the demand of high demanders." The demand game generated by this method is as follows. Each agent has a strategy (demand) space which is R_+ and his cost share as a function of the demand profile is computed by (1). The payoff is given by the utility function defined above. It should be noted that the serial cost sharing method (1) is defined for any arbitrary cost function. However, if we assume the cost function to be strictly⁶ convex (increasing marginal costs), then this demand game has very strong strategic properties. In this demand game the NE is unique, robust to coalitional deviations and the only rationalizable strategy profile. Moreover, this NE is the unique outcome of adaptive learning (Milgrom and Roberts, [16]).

Given the nice strategic and equity properties that the serial method enjoys in the homogeneous good setting, it is natural to look for the extension of the rule in more general settings. Friedman [5] studies the strategic properties of a class of such methods, which we describe in the next paragraph, and finds that these do enjoy nice strategic properties similar to serial cost sharing in the homogeneous goods case. He finds out the game induced by such methods is solvable by iterative elimination of *overwhelmed strategies*⁷ introduced in Friedman & Shenker [8].⁸

The natural extension of the serial method (1) to the heterogeneous case, where $C(q)$ is an arbitrary non-decreasing and continuously differentiable function of its n variables, which was introduced in Friedman and Moulin [7], is defined as follows.⁹ Consider a path¹⁰ γ^{SC} from 0 to q given by $\gamma^{SC}(t; q) = (te) \wedge q$, for $t \geq 0$, where $(p \wedge q)_i = \min\{p_i, q_i\}$ and $e = (1, 1, \dots, 1)$ is the unit vector in R^N . This path essentially follows the diagonal of the n -dimensional positive orthant until its coordinates are smaller than all the coordinates of the demand vector q . As soon as it meets the demand of some agent, it starts following the projection of the diagonal in the hyperplane where that coordinate is fixed at the demand in that coordinate and so on. Given such a path γ^{SC} the cost sharing mechanism is given by,

$$x_i^{SC}(q; C) = \int_0^\infty \partial_i C(\gamma^{SC}(t; q)) d\gamma_i^{SC}(t; q) \quad (2)$$

Here, $\partial_i C(p)$ is the partial derivative of C with respect to p_i evaluated at p . It is clear from (2) that the path relevant to an agent is independent of higher demands. Thus, the cost shares of agents are unaffected by small changes in the demands of higher demanding agents. Therefore, the externality is one sided (and, thus, acyclic). Intuitively, for this reason this mechanism enjoys nice strategic properties that we will see in Theorem 1. Moreover, for the same reason, the nice strategic properties are preserved if the γ^{SC} is replaced by any arbitrary continuous non-decreasing path $\phi(t; C) \wedge q$, where ϕ satisfies the following properties. For fixed C , ϕ is non-decreasing and continuous in t with $\phi(0; C) = 0$ and $\lim_{t \rightarrow \infty} \phi_i(t; C) > q^{\max}$ for all i . See figure 1 below for an example of such a ϕ .

This liberty of choosing the ϕ gives rise to a huge class of cost sharing methods called fixed path methods (FPM). There is an FPM corresponding to each fixed path ϕ that can be defined as follows.

$$x_i^\phi(q; C) = \int_0^\infty \partial_i C(\phi(t; C) \wedge q) d(\phi_i(t; C) \wedge q_i) \quad (3)$$

The path ϕ that uniquely defines a cost sharing method does not depend on q and thus is fixed in a sense. One example of a fixed path is the path which follows the edges of the n -orthotope (whose two diagonally opposite vertices are the demand

⁵An special case which is widely studied in this framework is the preference which is quasilinear in x_i and concave in q_i .

⁶strictness is not needed if the preferences of the agents are strictly convex.

⁷A strategy s_i for agent i is overwhelmed by strategy \tilde{s}_i with respect to S_{-i} if the best that agent i can get over S_{-i} by playing s_i is worse than the worst that he gets by playing \tilde{s}_i .

⁸Notice that this is stronger property than solvability in elimination of dominated strategies.

⁹Note that for the characterizations (e.g., [24], [13]) of the Moulin-Shenker method introduced in the next paragraph, more restrictions on the cost function C may be required. For example, [24] requires C to be twice continuously differentiable with bounded derivatives with no fixed costs whereas, [13] also requires C to be strictly increasing, and Lipschitz-continuity and boundedness of its partial derivatives.

¹⁰SC in the symbol underlines the point that this path corresponds to the generalization of Serial Cost (SC) sharing rule.

vector q and the zero vector 0) in some pre-decided order and this leads to the incremental methods. Notice that when the cost function is symmetric or when ϕ is independent of the cost function then the only symmetric FPM is the Friedman-Moulin method (2) defined by the path that is the diagonal of the positive orthant. Leroux [15] provides a justification of non-symmetric paths. However, symmetry is trivially satisfied when the cost function is not symmetric and we allow ϕ to be a function of C . This gives rise to a huge class of symmetric methods. Clearly, we will be sacrificing *additivity* in most of the cases but we can recover scale invariance and even stronger properties like *ordinality*¹¹ (see Sprumont [24]). The path that most closely follows the spirit of the serial method is the path that defines the Moulin-Shenker ordinal method discussed in (Sprumont [24]). This path which we will call ϕ^{MS} , is defined by the solution of the following differential equation

$$d\phi_i^{MS}(t; C)/dt = 1/\partial_i C(\phi^{MS}(t; C))$$

satisfying the boundary condition $\phi^{MS}(0; C) = 0$. This path has the property that on any point on the path the incremental cost generated by a small move along the path is shared equally among the agents not fully served. Other examples of FPMs can be generated by applying a FPM to any suitably normalized problem e.g. applying FPM to axially normalized problem (Friedman [5]). One seemingly natural FPM thus generated discussed in Friedman [5] is the use of diagonal path after axial normalization of the problem.

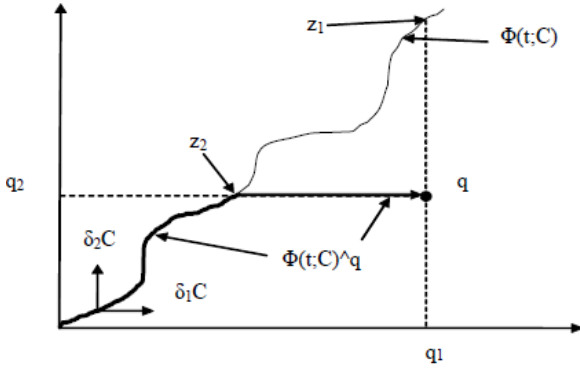


Figure 1: Fixed path method in two agent case

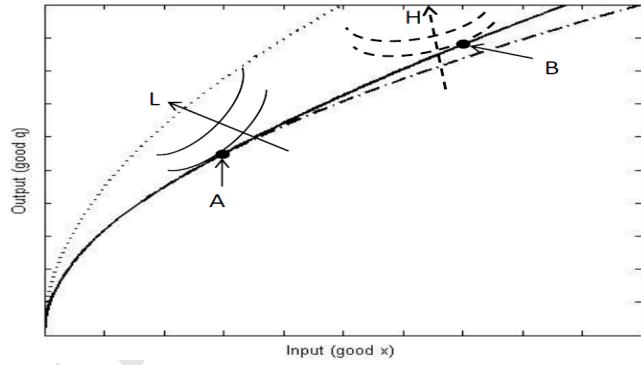


Figure 2: Serial SCF in two agent case

Given the set of agents N , utility profile $u = \{u_i\}_{i \in N}$, and a cost function C , a fixed path method $x^\phi(\cdot; \cdot)$ induces a cost sharing game $\Gamma(x, u)$. These induced games have a variety of strategic properties: uniqueness of NE, strong equilibria, uniqueness of the set of rationalizable outcomes, and convergence of adaptive learners. Friedman [5] shows these properties for fixed path methods by showing that the induced games are O-solvable which in turn implies all of these properties.

Theorem 1 (Friedman [5]): *Assume that the marginal cost ($\partial_i C(q)$) is strictly increasing in all variables, $x_i^\phi(\cdot; \cdot)$ is a fixed path method and that preferences, $u_i(q_i, x_i)$ are increasing in q_i , decreasing in x_i , and concave. Then the induced game is O-solvable.*

It should be noted that there can be paths that depend on q and we can use such paths to define "path methods" in a similar fashion as in (3). One prominent example of such a path method is the Aumann-Shapley method where the path is the ray joining the origin to the demand vector; thus for each demand there corresponds a path. More precisely, the path that generates the Aumann-Shapley method is given by $\phi^{AS}(t; C)(q) = tq$. We notice that this path is not a fixed path and the demand game generated by this method does not share the appealing strategic properties enjoyed by the FPMs. The Aumann-Shapley method in the homogeneous goods case is the proportional method. It has been shown in Watts [26] (see also Moulin [17] for detailed analyses) that uniqueness of NE is not guaranteed in the proportional demand games for general convex preferences and a sufficient condition has been shown to be the binormality of preferences. Moreover, as we will discuss in the next section, even when the NE is unique, this method doesn't share the strategic properties of the FPMs. Intuitively, this happens because a change in the demand by any agent changes the cost shares of all agents. For more on such path methods and the axiomatic characterization of methods generated by paths and more generally by convex combinations of paths, please refer to Friedman and Moulin [7].

4 Serial SCF and generalized serial SCF

We mentioned in the last section that if the production technology has increasing marginal costs and preferences are convex, then the serial rule (1) defined in the homogeneous goods case induces a game that admits a unique NE. A serial social choice function (SCF) for a fixed cost function C associates this unique NE allocation to the preference profile generating this game.

¹¹Ordinality is a stronger requirement than scale invariance. Scale invariance requires that the cost shares should be invariant to linear transformation of the demand profile whereas ordinality requires that it should be invariant to any arbitrary monotonic transformations, possibly non linear.

Figure 2 above demonstrates the serial mechanism (SCF) in a two-agent and two-goods economy where one good x is the input (horizontal axis) and the other good q is output (vertical axis). The production technology is decreasing returns to scale, i.e., the cost function is convex. The dotted curve is $c(q)$; the dash-dot curve is $c(2q)/2$; and the solid curve coincides with the dash-dot one until point A and then goes parallel to dotted curve. More precisely, the solid curve has two parts. The part below point A is the locus of points that are $1/2$ of some point on the dotted curve. The part above point A is the locus of points whose vector sum to the point A belongs to the dotted curve. The high valuation agent H (dashed indifference curve) is the agent whose MRS is higher for the output with respect to the input. The other agent is the low valuation agent L (solid indifference curve). The agents are required to report their utility functions and allocation is assigned according to the serial cost sharing rule. An informationally efficient way to implement the serial SCF as discussed in Moulin and Shenker [19] is as follows. In the first stage each agent reports a demand. One of the smallest demanders (call agent 1) demanding q_1 (say) gets q_1 and pays a cost share x_1 calculated by equation (1) for the demand profile (q_1, q_1, \dots, q_1) . This is equivalent to agent 1 in figure 2 optimizing on the dash-dot curve. In the second stage, all the agents except agent 1, report their demands (no smaller than q_1). A smallest demander (call agent 2) demanding q_2 (say) gets q_2 and pays a cost share x_2 given by the equation (1) for the profile $(q_1, q_2, q_2, \dots, q_2)$. This is equivalent to agent 2 optimizing on the solid curve in figure 2. Similarly, in the k th stage, all agents except agents 1 through $k - 1$ report their demands (at least as much as q_{k-1}) and the lowest demander will be named as agent k , served his demand q_k , and will be charged a cost share x_k corresponding to the equation (1) for the demand profile $(q_1, q_2, \dots, q_{k-1}, q_k, q_k, \dots, q_k)$.

The purpose of bringing the 2×2 case of serial SCF here is that the generalized serial SCF is defined very closely in the spirit of serial mechanism here. The three conditions below in the definition of generalized serial functions are linked to the following three observation in the above picture.

- 1) The opportunity set of L remains unaffected by changes in the preferences of H as long as H has higher valuation than L.
- 2) Owing to the convexity of the production function and preferences, there is a unique maximizer point A for L on his opportunity set given H and also B for H on her opportunity set given L.
- 3) Owing to no kinks in the solid curve and the dash-dot curve at point A, A remains the optimum point for L even after small changes in preference by H.

Generalized serial mechanism (SCF):

The generalized serial SCF is defined for an economy with n agents and m goods where n and m are greater than 1. Production technology \mathbf{P} is described by a $m - p$ dimensional smooth manifold that represents a technology where out of m -goods, p are inputs and $m-p$ are outputs. Set of alternatives A is the set of allocations to the agents in N which is feasible under \mathbf{P} . More formally, $A \equiv \{x \in R_+^{mn} | (\sum_{i=1}^n x_i) \in \mathbf{P}\}$. One example of such a set of alternatives where $m = 2$ and $p = 1$ is the set of allocations for the two agents in the above example which add up to a point on the dotted curve in figure 2. In this example, the production technology exhibits decreasing returns to scale where the input x can be interpreted as the cost of the output q . The set of admissible utility functions U_i for agent i contains the functions $u_i : R_+^m \rightarrow R$ that are continuous, non-decreasing in all dimensions, locally non satiated and quasi-concave. Linear utilities of agent i that constitute the subset \mathbf{L} of U_i will be considered isomorphic to $R_+^m - \{0^m\}$.

Definition 5 (Generalized Serial Function (Shenker) [23]) ¹² Consider a function $a : R_+^n \rightarrow A$ s.t., $\forall z \in R_+^n$ and $\forall \lambda \in \mathbf{L}$:

- (1) $z_i \leq z_j \Rightarrow a_i(z) = a_i(z_{-j}, s_j), \forall s_j \in [z_i, \infty)$,
 - (2) $\lambda \cdot a_i(z_{-i}, s_i)$ has a unique maximizer \bar{s}_i , $\forall i$,
 - (3) If \bar{s}_i is the unique maximizer of $\lambda \cdot a_i(z_{-i}, s_i)$ then \bar{s}_i is also the unique maximizer of $\lambda \cdot a_i(z'_{-i}, s_i) \forall z' \text{ s.t., } \forall j \neq i, MIN[z'_j, z_j] < \bar{s}_i \Rightarrow z'_j = z_j$.
- Such a function a is called a "generalized serial function."

The notation $a \cdot b$ means the inner product of the vectors a and b . The MIN function picks the minimum of the set. The 3rd point of the definition above captures the notion that the optimum in a coordinate remains unchanged as long as the bigger coordinates remain bigger. Let's denote by \mathbf{F} the set of all generalized serial functions. For a given utility profile u a function $a \in \mathbf{F}$ induces the normal form game $\Gamma(a; u) \equiv \langle N, \{S_i = R_+\}_{i \in N}, \{u_i(a_i(\cdot))\}_{i \in N} \rangle$ where N is the set of players, R_+ is the strategy space S_i for each player i , and the payoff function for player i is given by $u_i \circ a_i(\cdot)$. Such games possess a unique NE.

Lemma 1 : $\forall u \in U, \forall a \in \mathbf{F}; \Gamma(a; u)$ has a unique NE.¹³

Proof: The proof consists of two steps. In first step it is shown that there can not be more than one NE and then an explicit algorithm is given to construct an NE. A formal proof can be found in Appendix A.1 below.

¹²This function is similar to the one considered by Shenker but not exactly the same. The domain of the function a in Shenker is $(0, 1)$ whereas it R_+ in our definition. This difference, however, is not significant.

¹³Notice that this lemma is same as the lemma 1 in Shenker [23] (up to a minor change in the domain of the function a as noted).

Definition 6 (Generalized Serial Mechanism) ζ^a is a generalized serial mechanism (GSS) associated with $a \in \mathbf{F}$ if $\zeta^a(u) = a(z)$ where z is the unique NE of $\Gamma(a; u)$.

5 Secure implementability of generalized serial mechanisms and the fixed path mechanisms

Now we are ready to present our main result, which encompasses the result of Saijo et al. [22]

Theorem 2 : Any generalized serial mechanism (GSS) is securely implementable.

Proof: We show the secure implementability of GSS by showing that the GSS are strategyproof and that they satisfy the RP. Then by proposition 1 the desired result follows. Please refer to Appendix A.2 below for a complete proof.

Now we define a class of social choice functions called fixed path social choice functions based on fixed path cost sharing rules. Let's assume the conditions on the cost function and the preferences that were used in theorem 1. Then from Theorem 1 we know that there will be a unique NE in the game $\Gamma(x^\phi, u)$ induced by the cost sharing rule x^ϕ based on the fixed path ϕ .

Definition 7 A fixed path social choice function ξ^{x^ϕ} associates the allocation corresponding to the unique NE of the game $\Gamma(x^\phi, u)$ to the preference profile u .

The following theorem states that all such fixed path SCFs are securely implementable.

Theorem 3: Under the assumptions of Theorem 1, a fixed path social choice function ξ^{x^ϕ} is a special case of the generalized serial social choice function and thus is securely implementable.

Proof: The proof consists of explicitly constructing a generalized serial function a for every fixed path social choice function ξ^{x^ϕ} . We use two lemmas for proving the desired properties of such a . Please refer to Appendix A.3 below for a comprehensive proof.

At this time we would like to emphasize that the SCFs corresponding to path methods other than fixed path methods may not be securely implementable. One such method as we discussed above is the Aumann-Shapley method, which corresponds to the proportional method in the homogeneous goods case. To ensure the uniqueness of NE in the demand game, let's consider linear utilities (which are obviously binormal) given by $u_i(q_i, x_i) = b_i q_i - x_i$ and convex cost technology given by $c(y) = y^2/2$. Proportional cost shares are given by $x_i^{pr}(q) = \frac{q_i}{q_N} c(q_N)$, where $q_N = \sum_{i \in N} q_i$. Let's define the proportional SCF $\xi^{x^{pr}}$ that associates to every utility profile u , the unique NE of the demand game $\Gamma(x^{pr}, u)$. We notice that this SCF is not securely implementable. As a matter of fact it is not even strategyproof. To see this, consider a two-agent situation. Let the linear utilities of agents 1 and 2 be defined by the parameters b_1 and b_2 . Then, whenever b_i 's are close enough to ensure the active participation of both agents, the unique NE demand profile (q_1^*, q_2^*) is given by $q_i^* = \frac{4}{3}(b_i - \frac{b_j}{2})$ and the equilibrium cost shares turn out to be $x_i = \frac{4}{9}(b_i - \frac{b_j}{2}) + \frac{2}{3}b_j(b_i - \frac{b_j}{2})$, $i, j \in \{1, 2\}$. Therefore, the optimal report \bar{b}_i^* of agent i with true parameter b_i is given by $\bar{b}_i^* = \frac{3}{2}b_i + \frac{5}{4}b_j$ where \bar{b}_j is the report of agent j . Clearly, there are profitable manipulations of reports by agents. In particular, suppose $b_1 = b_2 = b$ and agent 1 reports truthfully, then the optimal report of agent 2 is $\frac{11}{4}b$.

We see that the FPMs are a special case of GSS. However, there are GSS that can not be represented by FPM. One trivial example is a constant SCF. Therefore, we conclude that GSS are more general than FPMs and have nice strategic properties.

6 Conclusion

In this paper, we have considered the environments where even the strong notions of implementation like "full implementation in dominant strategy equilibria (DSE)" or "full implementation in Nash equilibria (NE)" do not seem strong enough and the evidence shows that a more robust concept is needed. "Secure implementation" (Saijo et al. [22]) requires the existence of a mechanism that fully implements the SCF in NE and DSE. We show that a very broad class of SCFs is securely implementable in production economies. This result encompasses the work by Saijo et al. [22] in that we find much more general functions applicable to much more general environments to be securely implementable.

An interesting and challenging question in this framework is to characterize the class of SCFs that are securely implementable. Although there are characterization results for specific environments like group strategy proof mechanisms for binary goods (Juarez[10]) or two-agent characterizations under unique Nash equilibrium of profit sharing game (Leroux [14]), there is no such characterization for the general cost sharing mechanisms even under the classic concepts like implementation under DSE. Neither is there such a characterization of the general cost sharing problems under strategy proofness or group strategy proofness. Given the stronger notion of secure implementation, we hope to get such a characterization. We conjecture that *every smooth, non-constant, anonymous and securely implementable SCF belongs the set of GSS*.

A Proofs

A.1 Proof of Lemma 1

Step 1- Given any $u \in \mathbf{U}$ and any $a \in \mathbf{F}$; $\Gamma(a; u)$ can not have more than one NE.

Proof: Let z and z' be two distinct NE with $z = (z_1, z_2, z_3, \dots, z_n)$ and $z' = (z'_1, z'_2, z'_3, \dots, z'_n)$.

There must exist an element i such that $z_i \neq z'_i$ and $\min\{z_j, z'_j\} < \min\{z_i, z'_i\} \implies z_j = z'_j$.

Without loss of generality, say $z'_i < z_i$. But then $z'_i = \underset{s \in [0,1]}{\operatorname{argmax}} u_i(a_i(s, z_{-i})) = z_i$ which is a contradiction.

Step 2- Given any $u \in \mathbf{U}$ and any $a(z) \in \mathbf{F}$; The following algorithm generates a profile z which is a (the) NE of $\Gamma(a; u)$.

Algorithm:

1) Set $z = (1, 1, \dots, 1)$.

2) Define $s_i^1 = \underset{s \in [0,1]}{\operatorname{argmax}} u_i(a_i(s, z_{-i})), \forall i$.

3) Without loss of generality, let $s_1^1 = \min_{i \neq 1}\{s_i^1\}$. Set $z_1 = s_1^1$ and leave the other elements of z unchanged.

4) Define $s_i^2 = \underset{s \in [0,1]}{\operatorname{argmax}} u_i(a_i(s, z_{-i})), \forall i$.

5) Without loss of generality, let $s_2^2 = \min_{i \neq 2}\{s_i^1\}$. Set $z_2 = s_2^2$ and leave the other elements of z unchanged.

6) Repeat the process to update z_3, z_4, \dots, z_n .

Claim: The profile z obtained by the above algorithm is a NE of $\Gamma(a; u)$.

Proof:

Claim 1. If $s_i^i \leq s_{i+1}^{i+1}$ for all $i = 1, 2, \dots, n-1$, then z is a NE.

Proof: Straightforward from condition 3 in Definition 5 and the way z has been constructed.

Claim 2. $s_i^i \leq s_{i+1}^{i+1}$ for all $i = 1, 2, \dots, n-1$.

Proof:

Part1- $s_1^1 \leq s_2^2$. This holds because, $s_1^2 = s_1^1 = z_1$ (because 1 is solving the same optimization exercise) and $s_2^2 < s_1^1 \implies s_2^2 = s_1^1$ which contradicts the definition of s_1^1 .

Part2- If $s_i^i \leq s_{i+1}^{i+1}$ for all $i < k$ then $s_k^k \leq s_{k+1}^{k+1}$.

Proof: Notice first that $s_l^k = z_l = s_l^l$ for all $l < k$. This is true because of condition 3 in Definition 5. Now $s_{k+1}^{k+1} < s_k^k \implies$

$s_{k+1}^{k+1} = s_{k+1}^k$ which contradicts the definition of s_k^k . ■

A.2 Proof of Theorem 2

Strategyproofness of GSS follows from Theorem 7.2.3 in Dasgupta et al. [4], given our domain of preferences being monotonically closed and the fact that GSS is a single valued Nash implementable SCF.

We will prove the rectangularity property:

$$\forall u, \tilde{u} \in \mathbf{U}; \{u_i(\zeta^a(\tilde{u})) = u_i(\zeta^a(u_i, \tilde{u}_{-i})) \mid \forall i \in N\} \implies \zeta^a(\tilde{u}) = \zeta^a(u).$$

Proof:

Fix an arbitrary pair of utility profiles $u, \tilde{u} \in \mathbf{U}$.

Let $u_i(\zeta^a(\tilde{u})) = u_i(\zeta^a(u_i, \tilde{u}_{-i})) \mid \forall i \in N$.

Define, $NE(\Gamma(a; u_i, \tilde{u}_{-i})) = \tilde{z}^i$; $NE(\Gamma(a; \tilde{u})) = \tilde{z}$; $NE(\Gamma(a; u)) = z$. (Notice the notation; \tilde{z}^i is a vector and \tilde{z}_i is the i 'th component of the vector \tilde{z} . For example, \tilde{z}_k^i is the k 'th component of \tilde{z}^i .)

Step 1: $\tilde{z}^i = \tilde{z}$, $\forall i \in N$.

Proof: Let $\tilde{z}^i \neq \tilde{z}$ for some i .

Now, we must have an element k s.t. $\tilde{z}_k^i \neq \tilde{z}_k$ and $\min\{\tilde{z}_j^i, \tilde{z}_j\} < \min\{\tilde{z}_k^i, \tilde{z}_k\} \implies \tilde{z}_j^i = \tilde{z}_j$.

Case 1: $k \neq i$.

Without loss of generality, say, $\tilde{z}_k^i < \tilde{z}_k$.

$$\tilde{z}_k^i = \underset{s \in [0,1]}{\operatorname{argmax}} \tilde{u}_k(a_k(s, \tilde{z}_{-k}^i)) = \underset{s \in [0,1]}{\operatorname{argmax}} \tilde{u}_k(a_k(s, \tilde{z}_{-k})) = \tilde{z}_k \text{ which is a contradiction.}$$

Case 2: $k = i$.

Here there are two relevant cases,

Case 2.1: $\tilde{z}_i^i < \tilde{z}_i$.

Then we have,

$$\tilde{z}_i^i = \underset{s \in [0,1]}{\operatorname{argmax}} u_i(a_i(s, \tilde{z}_{-i}^i)) = \underset{s \in [0,1]}{\operatorname{argmax}} u_i(a_i(s, \tilde{z}_{-i}))$$

From property 1 in the definition of a , we must have the following.

$$a_i(\tilde{z}_i^i, \tilde{z}_{-i}^i) = a_i(\tilde{z}_i^i, \tilde{z}_{-i})$$

$$\text{or, } a_i(\tilde{z}^i) = a_i(\tilde{z}_i^i, \tilde{z}_{-i})$$

$$\text{Thus, } u_i(a_i(\tilde{z}^i)) = u_i(a_i(\tilde{z}_i^i, \tilde{z}_{-i})).$$

We also know, $u_i(\zeta^a(\tilde{u})) = u_i(\zeta^a(u_i, \tilde{u}_{-i})) \quad \forall i \in N$

$$\text{or, } u_i(a(\tilde{z})) = u_i(a(\tilde{z}^i)), \quad \forall i \in N$$

$$\text{or, } u_i(a_i(\tilde{z})) = u_i(a_i(\tilde{z}^i)), \quad \forall i \in N.$$

$$\text{Therefore, } u_i(a_i(\tilde{z})) = u_i(a_i(\tilde{z}_i^i, \tilde{z}_{-i})).$$

$$\text{In other words, } u_i(a_i(\tilde{z}_i, \tilde{z}_{-i})) = u_i(a_i(\tilde{z}_i^i, \tilde{z}_{-i})).$$

$$\text{But then, } \tilde{z}_i^i = \tilde{z}_i \text{ because } \tilde{z}_i^i \text{ is unique maximizer of } u_i(a_i(s, \tilde{z}_{-i}^i)) \text{ and } u_i(a_i(s, \tilde{z}_{-i})).$$

$$\text{Case 2.2: } \tilde{z}_i^i > \tilde{z}_i.$$

From property 1 in the definition of a , we must have the following.

$$a_i(\tilde{z}_i, \tilde{z}_{-i}) = a_i(\tilde{z}_i, \tilde{z}_{-i}^i)$$

$$\text{or, } a_i(\tilde{z}) = a_i(\tilde{z}_i, \tilde{z}_{-i}^i)$$

$$\text{Thus, } u_i(a_i(\tilde{z})) = u_i(a_i(\tilde{z}_i, \tilde{z}_{-i}^i)).$$

$$\text{We also know, } u_i(a_i(\tilde{z})) = u_i(a_i(\tilde{z}^i)), \quad \forall i \in N.$$

Therefore we have,

$$u_i(a_i(\tilde{z}^i)) = u_i(a_i(\tilde{z}_i, \tilde{z}_{-i}^i)).$$

$$\text{But then, } \tilde{z}_i^i = \tilde{z}_i \text{ because } \tilde{z}_i^i \text{ is unique maximizer of } u_i(a_i(s, \tilde{z}_{-i}^i)) \text{ and } u_i(a_i(s, \tilde{z}_{-i})).$$

□

Notice, the above step establishes the following property :

$$\tilde{u}_i(a_i(s, \tilde{z}_{-i})) \text{ and } u_i(a_i(s, \tilde{z}_{-i})) \text{ both are maximized at } \tilde{z}_i = \tilde{z}_i^i \text{ for all } i.$$

Let us call this property *property star*.

$$\text{Step 2: Claim: } \zeta^a(\tilde{u}) = \zeta^a(u).$$

Proof:

$$\text{Proving } a(z) = a(\tilde{z}) \text{ should be enough since, by definition } \zeta^a(\tilde{u}) = \zeta^a(u) \iff a(z) = a(\tilde{z}).$$

In fact, we will prove a stronger property, namely, $z = \tilde{z}$.

Suppose not and let $z \neq \tilde{z}$.

$$\text{Now, we must have an element } k \text{ s.t., } z_k \neq \tilde{z}_k \text{ and } \min\{z_j, \tilde{z}_j\} < \min\{z_k, \tilde{z}_k\} \implies z_j = \tilde{z}_j.$$

There can be two cases,

Case 1. $z_k > \tilde{z}_k$.

Then we get the following expression, where first and fourth equalities are from definition, second follows from the *property star* and third is due to the property 3 in the definition of function a .

$$\tilde{z}_k = \underset{s \in [0,1]}{\operatorname{argmax}} \tilde{u}_k(a_k(s, \tilde{z}_{-k})) = \underset{s \in [0,1]}{\operatorname{argmax}} u_k(a_k(s, \tilde{z}_{-k})) = \underset{s \in [0,1]}{\operatorname{argmax}} u_k(a_k(s, z_{-k})) = z_k \text{ and we reach a contradiction.}$$

Case 2. $z_k < \tilde{z}_k$.

Then we get the following expression, where first and fourth equalities are from definition, third follows from the *property star* and second is due to the property 3 in the definition of function a .

$$z_k = \underset{s \in [0,1]}{\operatorname{argmax}} u_k(a_k(s, z_{-k})) = \underset{s \in [0,1]}{\operatorname{argmax}} u_k(a_k(s, \tilde{z}_{-k})) = \underset{s \in [0,1]}{\operatorname{argmax}} \tilde{u}_k(a_k(s, \tilde{z}_{-k})) = \tilde{z}_k \text{ and we hit another contradiction}$$

to conclude the proof.

■

A.3 Proof of Theorem 3

We first present two lemmas which will be the key to the proof of Theorem 3 below.

Lemma 2 (Lemma 1 in Friedman [5]): Assume that marginal cost is strictly increasing in all variables and that x_i^ϕ is a fixed path method. Define $z_i(q_i) = \min\{t | \phi_i(t) \geq q_i\}$. Then:

(a) $x_i^\phi(q; C)$ is strictly increasing and strictly convex in q_i .

(b) $x_i^\phi(q; C)$ is non decreasing in q_j for all $j \neq i$.

(c) For all q and q'_j such that both $z_j(q_j)$ and $z_j(q'_j) \geq z_i(q_i)$ then $x_i^\phi(q; C) = x_i^\phi(q_{-j}, q'_j; C)$.

Lemma 3 (Lemma 2 in Moulin & Shenker [19]): Let $h_1(\lambda), h_2(\lambda)$ be two increasing and strictly convex functions from R_+ onto itself that coincide up to λ_0 :

$$h_1(\lambda) = h_2(\lambda) \quad \text{for all } \lambda \in [0, \lambda_0]$$

Then for every utility function u_i in U_i , the (unique) maximizers of $u_i(h_k(\lambda), \lambda)$ on R_+ , denoted by λ_k , $k = 1, 2$ are on the same side of λ_0 :

$$\lambda_1 \geq \lambda_0 \iff \lambda_2 \geq \lambda_0, \quad \lambda_1 = \lambda_0 \iff \lambda_2 = \lambda_0.$$

Proof of Theorem 3:

Fix a cost function C satisfying the assumptions of theorem 1. Let the domain of utility functions representing the preferences satisfying the assumptions be U .

Let the the set of alternatives be $A \equiv \{(q, x) : q \in [0, q^{\max}]^N, x \in R_+^N \text{ and } \sum_{i \in N} x_i = C(q)\}$. Consider a fixed path ϕ and the associated fixed path social choice function $\xi^{x^\phi} : U \rightarrow A$ which allocates the outcome corresponding to the unique NE of $\Gamma(x^\phi, u)$ to the preference profile u .

Consider $D_i = \{0\} \cup \{t \in R_+ \mid \phi'_i(t)_- \text{ is positive}\}^{14}$ and $z_i(q_i) = \min[t \mid \phi_i(t) \geq q_i]$ (see figure 1 above for such an example of z_i). We claim that a function $a : \times_{i \in N} D_i \rightarrow A$ which is defined as follows is a generalized serial function and the associated generalized serial SCF $\zeta^a = \xi^{x^\phi}$. Let $a_i(z) = (q_i(z), x_i^\phi(q(z)))$ for all i , where $q_i(z) = \phi_i(z_i)$ and $q(z) = (\phi_1(z_1), \phi_2(z_2), \dots, \phi_n(z_n))$. We will now prove the three required properties (Definition 5) of a using Lemma 2 and Lemma 3 above and the assumption on preferences.

First thing to notice is that even though the domain of a is not the same as in the original definition, the properties of a is retained exactly. Now we will show the above three properties one by one. To see that property 1 is true, notice that z_i uniquely defines $q_i(z) = \phi_i(z_i)$ which is independent of z_{-i} . Also, part (c) of the lemma 1 implies that $x_i^\phi(q(z)) = x_i^\phi(q(z'))$ $\forall z' \text{ st., } \forall j \neq i, \text{ MIN}[z'_j, z_j] < \bar{s}_i \Rightarrow z'_j = z_j$. Property 2 is a consequence of part (a) of lemma 1 and the linearity of preferences. We first notice that strict convexity of $x_i^\phi(q; C)$ in q_i and linearity of preferences which are increasing in q_i and decreasing in x_i ensures a unique maximizer q_i^* . But then, there will a unique z_i^* for this q_i^* by the definition of z_i . Property 3 is a bit more subtle and the proof is as follows. Consider two points z and z' in $\times_{i \in N} D_i$. Consider a coordinate i . Let, $\forall j \neq i, \text{ MIN}[z'_j, z_j] < \bar{s}_i \Rightarrow z'_j = z_j$. Consider $\lambda \in \{R_+ \times R_+\} \setminus \{(0, 0)\}$. Let $\bar{s}_i = \arg \max_{s_i \in D_i} \lambda \cdot a_i(z_{-i}, s_i)$ and $\tilde{s}_i = \arg \max_{s_i \in D_i} \lambda \cdot a_i(z'_{-i}, s_i)$. Let's call $\{a_j(z_{-i}, \bar{s}_i)\}_{j \in N} = \{(\bar{q}_j, \bar{x}_j)\}_{j \in N}$ and $\{a_j(z'_{-i}, \tilde{s}_i)\}_{j \in N} = \{(\tilde{q}_j, \tilde{x}_j)\}_{j \in N}$. From part (c) of lemma 2 we know $x_i^\phi(\bar{q}_{-i}, q_i; C) = x_i^\phi(\tilde{q}_{-i}, q_i; C)$ for all $q_i \in [0, \bar{q}_i]$. Also, we know from part (b) of lemma 2 that $x_i^\phi(\bar{q}_{-i}, \cdot; C)$ and $x_i^\phi(\tilde{q}_{-i}, \cdot; C)$ both are strictly convex in q_i . By the definition of \bar{s}_i it follows that $\bar{q}_i = \arg \max_{q_i \in [0, q^{\max}]} \lambda \cdot (q_i, x_i^\phi(\bar{q}_{-i}, q_i; C))$. But then from lemma 3 we must have $\bar{q}_i = \tilde{q}_i$. Finally to conclude the proof we notice that q_i being a one to one function of z_i implies that $\bar{s}_i = \tilde{s}_i$. Also, because of the way we have defined ζ^a and ξ^{x^ϕ} they coincide.

■

References

- [1] Bochet, O., Sakai, T., (2010). Secure implementation in allotment economies. *Games and Economic Behavior*, 68, 35-49.
- [2] Clarke, E.H., (1971). Multipart pricing of public goods. *Public choice*, 2, 19-33.
- [3] Cason, T., Saijo, T., Sjöström, T., Yamato, T., (2006). Secure implementation experiments: do strategy-proof mechanisms really work? *Games and Economic Behavior*, 57, 206-235.
- [4] Dasgupta, P. S., Hammond, P. J. and Maskin, E. S., (1979). The implementation of social choice rules: Some general results on incentive compatibility. *Review of Economic Studies*, 46, 185-216.
- [5] Friedman E. J., (2002). Strategic Properties of Heterogeneous Serial Cost Sharing. *Mathematical Social Sciences*, 44, 145-154.
- [6] Friedman E. J., (2004). Strong Monotonicity in Surplus Sharing. *Economic Theory*, 23, 643-658.
- [7] Friedman, E. and Moulin, H. (1999). Three Methods to Share Joint Costs or Surplus. *Journal of Economic Theory*, 87, 275-312.
- [8] Friedman, E.J., Shenker, S., (1998). Learning and implementation in the Internet. *Mimeo*, available from [www.orie.cornell.edu / ~friedman](http://www.orie.cornell.edu/~friedman).

¹⁴ $(\phi_i)'_-(t)$ is the left hand derivative of ϕ_i at t .

- [9] Fujinaka, Y., Wakayama, T. (2008). Secure implementation in economies with indivisible objects and money. *Economics Letters*, 100, 91–95.
- [10] Juarez, R. (2008). Group strategy proof cost sharing. *Mimeo, University of Hawaii*.
- [11] Kolpin, V., (1996). Multi-product serial cost sharing: An incompatibility with the additivity axiom. *Journal of Economic Theory*, 69, 227–233.
- [12] Koster, M., Tijs, S., and Borm, P. (1998). Serial Cost Sharing methods for multi-commodity situations. *Mathematical Social Sciences*, 36, 229–242.
- [13] Koster, M. (2007). The Moulin–Shenker rule. *Social Choice and Welfare*, 29 (2), 271–293.
- [14] Leroux, J. (2008). Profit sharing in unique Nash equilibrium: Characterization in the two-agent case. *Games and Economic Behavior*, 62, 558–572.
- [15] Leroux, J. (2007). Cooperative production under diminishing marginal returns: interpreting fixed-path methods. *Social Choice and Welfare*, 29, 35–53.
- [16] Milgrom, P., Roberts, J., (1991). Adaptive and sophisticated learning in repeated normal form games. *Games and Economic Behavior*, 3, 82–100.
- [17] Moulin, H. (1995). Cooperative Microeconomics: A Game Theoretic Introduction. *Princeton University Press. Princeton, New Jersey*.
- [18] Moulin, H., (2002). Axiomatic Cost and Surplus Sharing. Chapter 6 in the *Handbook of Social Choice and Welfare*. Volume 1, 290–357.
- [19] Moulin, H., and Shenker, S. (1992). Serial Cost Sharing. *Econometrica*, 60, 1009–1037.
- [20] Moulin, H., and Sprumont, Y. (2005). On Demand Responsiveness in Additive Cost Sharing. *Journal of Economic Theory*, 125, 1–35.
- [21] Moulin, H., and Sprumont, Y. (2007). Fair allocation of production externalities: recent results. *Revue d'Économie Politique*, 117, 7–37.
- [22] Saijo, T., Sjöström, T., Yamato, T., (2007). Secure implementation. *Theoretical Economics*, 2, 203–229.
- [23] Shenker, S. (1992). On the Strategy-proof and Smooth Allocation of Private Goods in Production Economies. *Mimeo, Xerox Research Center, Palo Alto*.
- [24] Sprumont, Y., (1998). Ordinal Cost Sharing. *Journal of Economic Theory*, 81, 126–162.
- [25] Satterthwaite, M., and Sonnenschein, H. (1981). Strategy-Proof Allocation Mechanisms at Differentiable Points. *Review of Economic Studies*, 48, 587–597.
- [26] Watts, A. (1996). On the Uniqueness of Equilibrium in Cournot Oligopoly and Other Games. *Games and Economic Behaviour*, 13, 269–285.

Highlights

- We show the possibility of Secure Implementation in a very general environment.
- Generalized Serial SCFs are shown to be Securely Implementable.
- These mechanisms include the well-known Fixed Path Methods as a special case.